Electromagnetic wave in free space

Maxwell's electromagnetic equation in free space (i.e. $\rho = 0, \sigma = 0, \mu = \mu_0, \epsilon = \epsilon_0$) are as follows

1. $\nabla \cdot \vec{E} = 0$ --- (1)
2. $\nabla \cdot \vec{H} = 0$ --- (2) $[\because B = \mu_0 H]$
3. $\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$ --- (3)
4. $\nabla \times \vec{H} = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$ --- (4)

Now taking curl on both sides of equation (3)

$$\nabla \times (\nabla \times \vec{E}) = -\mu_0 \frac{\partial}{\partial t} (\nabla \times \vec{H})$$

Using eq^n (4) on the right hand side.

$$\nabla \times (\nabla \times \vec{E}) = -\mu_0 \frac{\partial}{\partial t} \left(\epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

$$= -\mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

or $\nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E} = -\mu_0\epsilon_0 \frac{\partial^2\vec{E}}{\partial t^2}$

$[\because \nabla\times(\nabla\times\vec{A}) = \nabla(\nabla\cdot\vec{A}) - \nabla^2\vec{A}]$

For free space $\nabla\cdot\vec{E}=0$

or $\nabla^2\vec{E} - \mu_0\epsilon_0 \frac{\partial^2\vec{E}}{\partial t^2} = 0$

or $\nabla^2\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = 0$ $\left[\mu_0\epsilon_0 = \frac{1}{c^2}\right]$ – (5)

Taking curl on both sides of equation (4)

$\nabla\times\nabla\times\vec{H} = \epsilon_0 \frac{\partial}{\partial t}(\nabla\times\vec{E})$

or $\nabla(\nabla\cdot\vec{H}) - \nabla^2\vec{H} = \epsilon_0\frac{\partial}{\partial t}\left(-\mu_0\frac{\partial H}{\partial t}\right)$

$\nabla^2\vec{H} - \mu_0\epsilon_0\frac{\partial^2\vec{H}}{\partial t^2} = 0$ $[\because \nabla\cdot\vec{H}=0]$

$\nabla^2\vec{H} - \frac{1}{c^2}\frac{\partial^2\vec{H}}{\partial t^2}$ – – (6)

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = \frac{1}{\sqrt{8.854\times10^{-12}\,\text{farad/m}\times 4\pi\times10^{-7}}}$$

again $\frac{\mu_0}{4\pi} = 10^{-7}$ web/A m , $\frac{1}{4\pi\epsilon_0} = 9\times10^{9}$ m/farad

or $c = 3\times10^{8}$ m/sec.

Equations (5) & (6) represents three dimensional wave equation without damping travelling with velocity c. The $\vec{E}$ & $\vec{H}$ vectors progate in free space with velocity of c.